cal Stanton number (*1*). Using Equations (5) and (16) for the velocity and temperature profiles, respectively, and Equation (17) for $\eta$ one gets the final expression for the Stanton number:

$$N_{St}\, N_{Pr}^{(n+1)/(\delta_{n+1})} = \frac{f}{2} \qquad (18)$$

### DISCUSSION

The same final result can also be derived by relating the heat flux at the wall to the fictitious wall shear stress which can in turn be related to the actual wall shear stress. It should be noted that this type of approach is limited to the case where $\delta/\delta'$ is greater than unity. This must be the case since only for $\delta \geqq \delta'$ is there a real $u$ for every $u'$ within the fictitious boundary-layer thickness $\delta'$. This means that the Prandtl number must be 1 or greater. The error is small however for fluids like air ($N_{Pr} = .7$).

This analysis assumed the velocity profile and from this derived the temperature profile. It is seen that the two profiles are quite similar, with the temperature profile being characterized by the same exponent $n$ as the velocity profile. This has been borne out experimentally by Reynolds et al. (*2*) who measured velocity and temperature profiles in the turbulent boundary layer. They found the value of the exponent $n$ to be the same for both the velocity and temperature distributions.

As can be seen the Prandtl number dependence of the Stanton number indicated in this analysis is very insensitive to the value of $n$. For $n = 7$ the exponent is 2/9, while for $n = 5$ it is 3/13.

The only experimental data known to the authors with Prandtl numbers very different from unity are those published recently by Ede and Saunders (*3*). Heat transfer coefficients were compared for water at temperatures such that the Prandtl number was 4.9 and 6.7. Although the Colburn analogy appeared to provide an adequate basis for comparing the effect of Prandtl number variation, scattering of the data make the conclusions somewhat tentative. In addition the effect of an unheated starting length was a part of the investigation, and the effect of this variable complicates the interpretation of the final results.

In a recent investigation Reynolds (*2*) studied incompressible, turbulent boundary layer heat transfer past a flat plate. This work (with air) indicated a Prandtl number dependence of the Stanton number substantially less than the usual Colburn type of correlation. A Prandtl number exponent of 0.4 was used as compared with 0.67 called for by the Colburn analogy.

### NOTATION

$C$ = local concentration
$C_1, C_2$ = constants
$C_p$ = heat capacity
$E_D$ = eddy diffusivity of mass
$E_h$ = eddy diffusivity of heat
$E_m$ = eddy diffusivity of momentum
$f$ = friction factor
$h$ = heat transfer coefficient
$k$ = thermal conductivity
$l$ = Prandtl mixing length
$n$ = exponent in empirical turbulent velocity profile
$N_{Pr}$ = $\nu/\alpha$
$N_{Re\,x}$ = $xU/\nu$
$N_{St}$ = $h/C_p\rho U$
$q''_w$ = wall heat flux
$t$ = local temperature
$t_w$ = wall temperature (constant)
$t_\infty$ = free stream temperature (constant)
$u$ = longitudinal velocity component
$u'$ = fictitious longitudinal velocity component
$U$ = free stream velocity
$U'$ = fictitious free stream velocity
$v$ = transverse velocity component
$v'$ = fictitious transverse velocity component
$x$ = longitudinal distance measured from front of plate
$y$ = transverse distance measured from surface of plate

#### Greek Letters

$\alpha$ = thermal diffusivity
$\delta$ = momentum boundary-layer thickness
$\delta'$ = fictitious momentum boundary-layer thickness or actual thermal boundary-layer thickness
$\eta$ = $\delta'/\delta$
$\nu$ = kinematic viscosity
$\rho$ = density
$\tau_w$ = wall shear stress

### LITERATURE CITED


1. Eckert, E. R. G., and R. M. Drake "Heat and Mass Transfer," 2 ed., McGraw-Hill, New York (1959).
2. Reynolds, W. C., W. M. Kays, and S. J. Kline, *Natl. Aeronaut. Space Administration Memo 12-1-58W*.
3. Ede, A. J., and O. A. Saunders, *Proc. Inst. Mech. Engrs.*, **172**, 23 (1958).


## CHEMICAL ENGINEERING PROGRESS SYMPOSIUM SERIES ABSTRACTS

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### HEAT TRANSFER—BUFFALO, Vol. 57, No. 32, 1961.

**Quantitative Evaluation of the Effect of Edge Losses and Contact Resistances in the Determination of Thermal Diffusivity of Solid Materials by an Unsteady State Method**, Arthur A. Armstrong and K. O. Beatty. Thermal diffusivity was determined from time-temperature data. Charts were constructed by relating the temperature at two points in the sample at steady state for various values of the two parameters. The thermal diffusivity was then calculated from the time-temperature data and the values of the two parameters. **On Unsteady State Heat Transfer in a Hollow Cylinder or Sphere**, Warren W. Clauson. This paper describes methods that are useful in the solution of unsteady state heat conduction problems for composite hollow cylinders or spheres. These methods are applicable to problems occurring in many industries; the chemical, missile, aircraft, and power industries are a few examples. A solution may be obtained using graphical or numerical techniques. **Some Aspects of the Melting Solution for a Semi-infinite Slab**, Manfred Altman. It is shown that the general nonlinear problem reduces to a quasilinear one in a region surrounding the peak of the heat-flux curve under certain conditions. Analogue solutions are presented for certain generalized cases. Approximate analytical solutions are presented which are based on the linearized equations resulting from the integral method. **Analysis of Transient Ablation and Heat Conduction Phenomena at a Vaporizing Surface**, R. G. Fledderman and H. Hurwicz. An exact analysis of heat and mass transfer in an ablating three-phase system is made. Account is also taken of internal heat radiation and of the results of steady state ablation theory. An approximate method based on effective heat of ablation and constant surface temperature is also developed. **Local Radial Effective Conductivity and the Wall Effect in Packed Beds**, R. F. Baddour and C. Y. Yoon. Theoretical and empirical correlations are proposed for the estimation of static-bed conductivities, and theoretical treatment is given to the cases of a solid temperature significantly different from that of fluid. **The Relation Between the Transfer Coefficient and Thermal Fluctuations in Fluidized-Bed Heat Transfer,** H. S. Mickley, D. F. Fairbanks, and R. D. Hawthorn. It is hypothesized that the periods of zero coefficient occur when gas bubbles pass the heater surface, and that the ensuing jumps in coefficient are due to the sudden contacting of packets of solid with the surface. The assumption of unsteady state conduction of heat with packets of quiescent solid is shown, when combined with the average observed frequency and with the fraction of time of zero coefficient, to lead to surprisingly good quantitative predictions of the observed time-average coefficients. **Heat Transfer with a Flowing Fluid Through Porous Media,** Don W. Green and Robert H. Perry. The numerical solutions to the general equations presented in this paper are used to determine the physical limits within which these assumptions are justified. The results of the computation are applicable to an idealized model of a thermal oil-recovery process and to packed-bed chemical reactors. **A Review of Fluid-To-Particle Heat Transfer in Packed and Moving Beds,** Thomas G. Bowers and Harold Reintjes. The data available in the literature are studied together with heat transfer results obtained by the authors. The equations are restricted to solids with the same general particle characteristics as sized gravel, refractory pebbles, and anthracite coal briquettes. **Dimensional Analysis and Natural Circulation,** J. D. Hellums and S. W. Churchill. Methods of determining the minimum number of dimensionless groups and of reducing the number of independent variables are described. These techniques are illustrated for the classical problem of natural convection from a vertical plate. **Correlations of Convective Heat Transfer in Confined Horizontal Layers,** J. L. O'Toole and P. L. Silveston. Correlations of heat transfer through horizontal layers heated from below have been developed from a large body of published and unpublished data by means of step-by-step multiple regression. Equations are proposed for the three distinct natural convection regimes, initial or creeping, laminar, and turbulent, which have been observed. **High-Intensity Natural-Convection Heat Transfer Near the Critical Point,** Charles F. Bonilla and Leon A. Sigel. It was found that the usual natural-convection correlations hold for intense turbulent natural convection in the critical and supercritical regions as they do for turbulent natural convection in the subcritical region. A new type of natural-convection regime which limits the further rise in heat transfer coefficient was observed at very high Rayleigh numbers. **Rates of Heat Transfer from Short Sections of an Isothermal Pipe,** A. A. Faruqui and J. G. Knudsen. Average heat coefficients were measured in short sections of a heated pipe with various entrance configurations. An equation is developed to relate the Nusselt number for a particular test section, the orifice or nozzle type, and the distance upstream from the test section. **Heat Transfer and Pressure Drop in Two-Phase Flow,** Kenneth E. Lunde. This paper is largely concerned with a correlation of two-phase, two-component heat transfer on the basis of a physical model, but it includes an extension to vaporization processes and two-phase pressure drop. **Analysis and Measurement of Flow Oscillations,** E. R. Quandt. This paper presents the results of an analytical and experimental investigation of flow instabilities. **Heat Transfer, Burnout, and Pressure Drop for Water in Swirl Flow Through Tubes with Internal Twisted Tapes,** W. R. Gambill, R. D. Bundy, and R. W. Wansbrough. Advantages of swirl-flow tubular fuel elements for nuclear reactor applications are discussed. **The Heat, Mass Transfer Characteristics of Evaporative Coolers,** Robert O. Parker and Robert E. Treybal. A new mathematical model for evaporative coolers has been devised in which the performance is described in terms of two transfer coefficients. The differential equations have been integrated to yield results useful in the design and testing of these exchangers. **Condensing Coefficient Inside a Horizontal Tube Near Atmospheric Pressure,** John A. Myers and Harold F. Rosson. An experimental investigation was conducted to determine the effect of the presence of liquid condensate on the heat being transferred during the condensation of a pure substance inside a horizontal tube. Two theoretical equations are developed which describe the effect of condensate flow rate on the condensing coefficient. **On the Mechanism of Subcooled Nucleate Boiling, Part I: Preliminary Considerations,** S. G. Bankoff. A sequential rate process model for heat transfer in forced-convection subcooled nucleate boiling is proposed. Expressions are deduced for the quenching heat flux and for the mean amplitude of the velocity fluctuation in the liquid between the bubbles. **On the Mechanism of Subcooled Nucleate Boiling, Part II: Sequential Rate Process Model,** S. G. Bankoff. In this part temperature distributions through the single-phase turbulent core are calculated for Gunther's data. **The Effect of Trace Additives on the Heat Transfer to Boiling Isopropanol,** Thomas Dunskus and J. W. Westwater. A study was made of the effect of eleven organic substances of high molecular weight on the heat transfer to boiling isopropanol in the nucleate, transition, and film-boiling regimes. **Nucleate-Boiling Studies with Aqueous Thorium Oxide Slurries,** David G. Thomas. Nucleate-boiling heat transfer measurements were made with aqueous thorium oxide slurries. No phenomena were observed which could be attributed to the effect of the solid particles on the gross physical properties of the slurry. **Effects of Acceleration on Nucleate Pool Boiling,** Charles P. Costello and William E. Tuthill. This paper presents the results of an investigation undertaken to determine the effects of accelerations produced by centrifugal effects on pool boiling heat transfer to distilled water. The investigation was instigated to better understand the mechanisms of boiling heat transfer, as well as to obtain data for accelerating systems. **An Experimental Study of Partial Film Boiling Region with Water at Elevated Pressures in a Round Vertical Tube,** J. B. McDonough, W. Milich, and E. C. King. The experimental techniques by which the data were obtained and a description of the experimental loop, test section, sample reduction method, and analysis of experimental errors are described.

# Computer Program Abstracts

**Readers of the *A.I.Ch.E. Journal* who are interested in programing for machine computation of chemical engineering problems will find in each issue of *Chemical Engineerng Progress* abstacts of programs submitted by companies in the chemical process industries. Collected by the Machine Computation Committee of the A.I.Ch.E., these programs will be published as manuals where sufficient interest is indicated. The following abstracts have appeared this year:**

**CEP (June, 1961), p. 82**

Diffusion Coefficients by the Guoy Method (075)

Determination of the Constants of the Callendar-Van Dusen Equa-